HCC MATH DAYS

Math 1314 Final Exam Review Problems

Updated: 03/21/22

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1. Solve the equation by factoring and applying the zero product property.

   (a) $3x^2 + 14x - 49 = 0$

   (b) $24x^2 + 6 = 24x$

   (c) $(n+5)(n-7) = 28$

   (d) $10m(m+3) = 3m - 5$

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2. Use the Square Root Property to solve the equation.

   (a) $16x^2 = 17$

   (b) $(k+6)^2 = 28$

   (c) $12x^2 + 48 = 0$

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3. Determine the value of n that makes the polynomial a perfect square. Then write the polynomial as the square of a binomial.

   (a) $p^2 + 22p + n$

   (b) $u^2 - 19u + n$

   (c) $x^2 - \dfrac{2}{3}x + n$

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4. Solve by completing the square.

   (a) $x^2 + 14x - 5 = 0$

   (b) $8x^2 + 3x - 32 = 0$

   (c) $4y^2 + 8y = 11$

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5. Use the quadratic formula to solve the equation

   (a) $2x(x+3) = -1$

   (b) $(3x-2)(x-1) = -3$

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6. Solve the equation using any method

$$\frac{5}{x-4} - \frac{8}{x+1} = \frac{34}{x^2-3x-4}$$

7. Solve for the indicated variable.

   (a) $A = \pi r^2 h$ for $r > 0$.

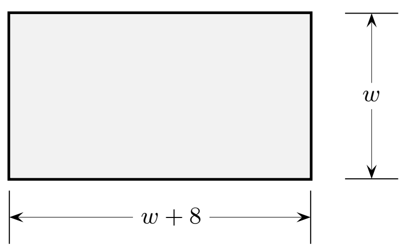
   (b) $kw^2 - cw = r$ for w. Assume $k > 0$.

8. Solve the equations. For each equation, find the ***sum of the solutions***.

   (a) $\sqrt{6x+7} - 2x = 3$

   (b) $\sqrt{2x+3} - \sqrt{x-2} = 2$

9. A rectangle has an area of 105 yds^2. The length of the rectangle is 8 yds more than its width w. Find the ***perimeter*** P of the rectangle.



10. The length of the longer leg of a right triangle is 14 ft longer than the length of the shorter leg x. The hypotenuse is 6 ft longer than twice the length of the shorter leg. Find the dimensions of the triangle.

11. Solve the equation $4x^{2/3} - 9x^{1/3} = 9$. Find the ***product*** of the solutions.

12. Solve the equation $(x-3)^4 - 5(x-3)^2 + 4 = 0$. Find the ***product*** of the solutions.

13. Solve the *absolute-value* equations. For each equation, find the ***sum*** S of the solutions.

   (a) $|8x-3| - 12 = 4$.

   (b) $|5x+4| = |x+9|$.

14. Solve the following inequalities. Write the solution sets in interval notation.

   (a) $(x-4)(5x-8) > 0$

   (b) $\dfrac{(x-4)(5x-8)}{x-15} < 0$

   (c) $\dfrac{x-4}{x-15} < 2$

15. Solve the absolute value inequalities. Write the solution sets using interval notation.

    (a) $|5x - 12| - 4 \geq 20$

    (b) $|4x - 9| < 10$

16. Let $P(3, -8)$ and $Q(-2, 5)$ be given points.

    (a) Use the ***distance formula*** to find the exact distance between P and Q.

    (b) Find the ***midpoint*** of the line segment whose endpoints are the given points P and Q.

17. Write the given equation in the form $(x-h)^2 + (y-k)^2 = r^2$. Identify the center and radius.

    $2x^2 + 2y^2 + 16x - 20y + 50 = 0$

18. The endpoints of the diameter of a circle are $(-2, 3)$ and $(-10, 9)$. Write an equation of this circle in standard form and identify its center and radius.

19. Find an equation for the line with the given property. Write the equations in slope-intercept form.

    (a) Perpendicular to the line $x - 5y = 3$ and containing the point $(5, 3)$.

    (b) Parallel to the line $2x - 5y = 3$ and containing the point $(5, 3)$.

    (c) Containing the points $(5, 3)$ and $(-1, 8)$.

20. Find the domain of each function. Write the domain using interval notation.

    (a) $f(x) = \dfrac{5x-4}{x+3}$ (b) $g(x) = \dfrac{5x}{\sqrt{x+3}}$ (c) $h(x) = \dfrac{x-4}{|x+3|}$

21. Determine if each function is ***even***, ***odd***, or neither.

    (a) $f(x) = -x^5 + x^3$ (b) $g(x) = x^2 - |x| + 1$ (c) $h(x) = 5x^2 + 3x$

22. Determine if the graph of each equation is ***symmetric*** with respect to the x-axis, y-axis, origin, or none of these.

    (a) $y = -x^2 + 3$ (b) $x = -|y| + 4$ (c) $y = x^2 + 5x + 1$ (d) $x^2 - y^2 = 5$.

23. Define the function f by

$$f(x) = \begin{cases} 10x + 2 & \text{if } x \geq 6, \\ 5x - 6 & \text{if } -3 \leq x < 6, \\ x^2 - 2 & \text{if } x < -3. \end{cases}$$

    Evaluate:

    (a) $f(-4) + f(5)$ (b) $f(0)$ (c) $f(6)$

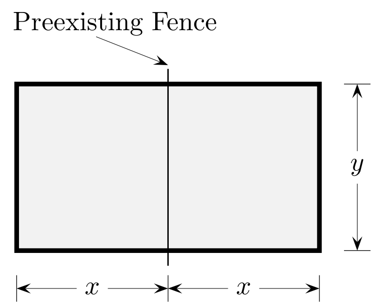
24. Solve the problem.

    If $f(x) = \dfrac{3x - B}{x - A}$, $f(3) = 0$, and $f(-6)$ is undefined, what are the values of A and B?

25. Mr. Green Jeans is planning to build two identical chicken coops on opposite sides of a preexisting fence. His chicken coops are to be built adjacent to one another from **120** feet of fencing. (See the figure to the right.)

    (a) What dimensions x and y should be used to maximize the area of an individual coop?

    (b) What is the maximum area of an individual coop?



26. Determine the ***end behavior*** of the graph of each polynomial function given below.

    (a) $f(x) = -4x^5 + 6x^3 + 2x$ (b) $g(x) = 5x(2x - 3)^3(x + 2)^2$ (c) $h(x) = -8x^4 - 5x^3 - 1$

27. Evaluate the function for the indicated value, then simplify as much as possible.

    $f(x) = x^2 - 3x + 5$. Find $f(x + 1)$.

28. Find $\dfrac{f(x + h) - f(x)}{h}$ for the following function f.

    $f(x) = x^2 - 3x + 5$.

29. For the given functions f and g, find $(f \cdot g)(x)$.

    $f(x) = \dfrac{x - 1}{x^2 - 25}$, $g(x) = \dfrac{x + 5}{1 - x}$

30. For the given functions f and g, find $\left(\dfrac{f}{g}\right)(-2)$.

    $f(x) = -6x + 1$, $g(x) = x^2$

31. For the given functions f and g, find and simplify the ***composite*** functions.

    (a) $(f \circ g)(x)$.

    (b) $(g \circ f)(x)$.

    $f(x) = 6x^2 - x + 1$, $g(x) = x - 3$

32. Describe, with graph transformations, how the graph of $f(x) = (x - 2)^2 + 5$ relates to the graph of the parent function $g(x) = x^2$.

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33. Form a polynomial $f(x)$ with real coefficients having the given degree and zeros.

    degree: 4; zeros: $-1, 2$, and $1 - 2i$.

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34. Use the **Rational Zeros Theorem** to find all the real zeros of the polynomial function. Use the zeros to factor f over the real numbers.

    $f(x) = 4x^3 - 11x^2 - 6x + 9$

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35. Use the **Rational Zeros Theorem** to list all the possible rational zeros of the polynomial function. Do not find the actual zeros.

    $f(x) = 6x^4 + 3x^3 - 4x^2 + 2$

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36. Solve the problem.

    Find m so that $x + 4$ is a factor of $5x^3 + 18x^2 + mx + 16$.

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37. Find the ***average rate of change*** of the function f from $x_1 = 1$ to $x_2 = 5$, where $f(x) = 4x^2 - 6x + 1$

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38. Find the ***vertical asymptotes*** (VA) and ***horizontal asymptotes*** (HA), if any, for each function.

    (a) $f(x) = \dfrac{x-1}{x^2-25}$ (b) $g(x) = \dfrac{3x-7}{5x+12}$ (c) $h(x) = \dfrac{x^2-4}{x+1}$

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39. Find the zeros of the polynomial function and state the multiplicity of each zero.

    $f(x) = 10x(x-1)^4(x+3)^2$

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40. If the vertex of the parabola $y = 4x^2 - 5x + 10$ is the point (h, k), what is the value of k?

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41. Find the inverse function of the function f, if it exists, where

    (a) $f(x) = -12x + 5$.

    (b) $f(x) = (x-1)^3 + 4$.

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42. Use the Remainder Theorem and synthetic division to find $f(k)$, where

    $k = \frac{1}{2}$ and $f(x) = 4x^3 - 7x^2 + 5x - 3$

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43. Divide the polynomials using *synthetic division.*

    $\dfrac{3x^4 - 5x^2 + 15x + 2}{x - 2}$

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44. Divide the polynomials using *long division.*

$$\frac{3x^4 - 2x^3 - 5x^2 + 15x + 2}{x^2 - 3}$$

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45. Solve the nonlinear system . Provide the product P of the y-values of the solutions and the sum S of the x-values of the solutions.

$$x^2 - xy = 20$$
$$x - 2y = 3$$

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46. Solve the system of equations using Gaussian elimination.

$$x - 3y - 2z = 0$$
$$2x - 7y - 6z = 7$$
$$4x + 5y + 2z = 1$$

Then compute the ***sum*** $S = x + y + z$ of the solution (x, y, z) of the system.

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47. Write the expression $\log_3\left(\frac{r\sqrt[3]{ab}}{c^5}\right)$ as a sum, difference, or product of logarithms.

Assume that all variables represent positive real numbers.

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48. Suppose $\log_x 8 = B$, where B is a positive real number and $x > 0$. Solve for x as a function of B.

Find a value of B such that the solution x of the equation $\log_x 8 = B$ is a positive integer.

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49. Let $f(x) = \log_5(x + 3)$.

(a) Write the domain and range of f in interval notation.

(b) Determine the vertical asymptote of f.

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50. Solve the logarithmic equation $2 + \log_3(2x + 5) - \log_3(x) = 4$.

If the reciprocal of the solution is written as a reduced fraction $\frac{n}{m}$ (where n and m are integers whose greatest common factor is 1 and where $m > 0$), what is the value of m?

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51. Solve the logarithmic equations. For each equation, find the sum S of all solutions.
(Note: If there is only one solution $x = a$ for a given equation, then $S = a$ for that equation.)

(a) $\log_3(x + 5) + \log_3(x - 3) = 2$.

(b) $\log_2(x - 4) + \log_2(10 - x) = 3$

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52. Solve the exponential equations.

    (a) $4e^{(3x+2)} = 2.$

    (b) $7^{(3x+2)} - 15 = -3.$

    For each equation, express the solution set in terms of the natural logarithm.

53. Solve the exponential equation.

    $16^{(3x+2)} = 4^{(5x-8)}$

54. Solve for x.

    $\begin{vmatrix} x & 5 \\ -2 & 8 \end{vmatrix} = 12$

55. Use the given matrices to compute the given expression.

    Let $\mathrm{M} = \begin{bmatrix} 5 & 6 \\ -2 & 0 \end{bmatrix} \qquad \mathrm{N} = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$

    Find $4\mathrm{M} - 3\mathrm{N}$.